

Technical Note

Nodal analysis of a Stirling engine with concentric piston and displacer

H. Karabulut, H.S. Yücesu, C. Çınar*

*Department of Mechanical Technology, Faculty of Technical Education, Gazi University,
Besevler, 06500 Ankara, Turkey*

Received 15 May 2005; accepted 20 December 2005
Available online 7 February 2006

Abstract

To reduce the external volume of Stirling engines and to increase the specific power per unit volume, a novel mechanical arrangement is used where the power cylinder is concentrically situated inside the displacer cylinder. The inner heat transfer surface requirement and the thermodynamic performance characteristics are predicted preparing a nodal analysis in FORTRAN, where the inner volume of the engine is divided into 103 cells. Variation of the temperature in cells is calculated using the first law of thermodynamics, given for unsteady open systems, after arranging the enthalpy inflow and outflow terms. Volumes of cells are calculated using kinematic relations devised for the driving mechanism.

The analysis indicates that the heats received from and delivered to the regenerator are not equal to each other. Therefore, the ends of the regenerator should be coupled with a heater and a cooler. The maximum thermal efficiency appears at the minimum mass of working fluid as the minimum thermal efficiency appears at the maximum mass of working fluid. The work increases up to a certain value of working fluid and then decreases. The thermal efficiency increases until a certain value of regenerator area and then decreases as well. Fluid temperature in the hot volume and cooler differs from the wall temperature at significant rates.

© 2006 Elsevier Ltd. All rights reserved.

Keywords: Gamma-type Stirling engine; Nodal analysis; Concentric piston and displacer

*Corresponding author. Tel.: +90 312 2126820/1897; fax: +90 312 2120059.
E-mail address: cancinar@gazi.edu.tr (C. Çınar).

Nomenclature

A	heat transfer area (m^2)
E	enthalpy in or outflow (J/kg)
C_V	constant volume specific heat ($\text{J kg}^{-1} \text{K}^{-1}$)
C_P	constant pressure specific heat ($\text{J kg}^{-1} \text{K}^{-1}$)
h	heat transfer coefficient ($\text{W m}^{-2} \text{K}^{-1}$)
ℓ_d	length of displacer arm (m)
ℓ_p	length of piston arm (m)
m	mass (kg)
p	pressure (Pa)
Q	heat (J)
R	gas constant ($\text{J kg}^{-1} \text{K}^{-1}$)
R_d	radius of displacer pin on crankshaft (m)
R_p	radius of piston pin on crankshaft (m)
s_h	instantaneous length of hot volume (m)
s_c	instantaneous length of cold volume (m)
s_e	instantaneous length of expansion cylinder (m)
T	temperature (K)
t	time (s)
V	volume (m^3)
W	work (J)
η	thermal efficiency (%)
θ	crankshaft angle (rad)
ω	angular velocity (rad s^{-1})

Subscripts

c	cold
d	displacer
e	expansion
h	hot
i	cell counter
j	time step counter
p	piston
r	regenerator
t	total
w	wall

1. Introduction

Energy conversion from radiation to electricity is performed by means of photovoltaic solar cells and heat engines [1,2]. For photovoltaic cells, efficiencies up to 18% are reported [3]. The efficiency of heat engine conversion systems rises to 33% depending on the quality of technology used [4].

With respect to their thermodynamic principles, heat engines are classified into two groups: Stirling and Ericsson. In the theoretical cycle of the Stirling engine, the working fluid is compressed at constant temperature, heated at constant volume, expanded at constant temperature and cooled at constant volume. In the theoretical cycle of the Ericsson engine, heating and cooling of the working fluid is carried out in a manner different from that of the Stirling engine at constant pressure. In practice, thermodynamic cycles of both engines are performed by means of similar driving mechanisms and the practical cycles of both engines are similar. Therefore, the term ‘Stirling engine’ is generally used to represent all externally heated engines [5,6].

Despite the fact that Stirling engines have been investigated for 200 years, their use today is in energy investigation projects. The main disadvantages of Stirling engines are their large volume and weight, low compression ratio and leakage of working fluid from the engine inner volume. To increase the specific power of Stirling engines, a number of methods have been developed, such as using hydrogen or helium as working fluid at high charge pressure, increasing the temperature difference between hot and cold sources, increasing the internal heat transfer coefficient and heat transfer surface and using simple mechanical arrangements, e.g. a free piston Stirling engine [7–9].

With respect to their mechanical arrangements, Stirling engines are classified into three groups: alpha, beta and gamma [1,5]. In the alpha type of mechanical arrangement, the thermodynamic cycle is performed by means of two pistons working in separate cylinders: one is held at a high temperature and the other at a low temperature. In the beta type, the cycle is performed by means of a power piston and a displacer both working in the same cylinder. In most of the beta-type engines manufactured so far, the free end of the cylinder is held at a high temperature; however, the free end of the cylinder may also be held at a low temperature. In the gamma type, the cycle is performed by means of a power piston and a displacer working in separate cylinders. The cylinder in which the piston works is held at a high temperature. One end of the displacer cylinder is held at a low temperature while the other end is held at a high temperature. Due to their manufacturing simplicity, durability, high mechanical efficiency and self-pressurization feature, the gamma-type Stirling engines are more attractive [1,5,7,8].

In thermodynamic calculations of Stirling engines, isothermal analytic, isothermal nodal, adiabatic and nodal analyses are used [10–13]. In this investigation, a novel mechanical arrangement having comparatively smaller volume and weight is designed and theoretically analyzed using nodal analysis.

2. Mechanism and mathematical modeling

The mechanical arrangement is illustrated in Fig. 1. The displacer works in an annulus gap constituted by two cylinders situated concentrically. The outer surface of the annulus gap functions as a regenerator; therefore, axial slots are grooved.

The displacement of the working fluid between cold and hot spaces takes place through the slots. The displacer is a thin shell annulus cavity. The power piston works in the inner cylinder. The hot volume above the displacer is connected to the expansion volume. The connection of these two volumes is made again by means of slots functioning as the heat transfer surface. The cold volume below the displacer is also connected to the regenerator by means of slots functioning as the cooler. The convective heat transfer coefficient on the slot surfaces is calculated assuming that the flow through the slot is a fully developed

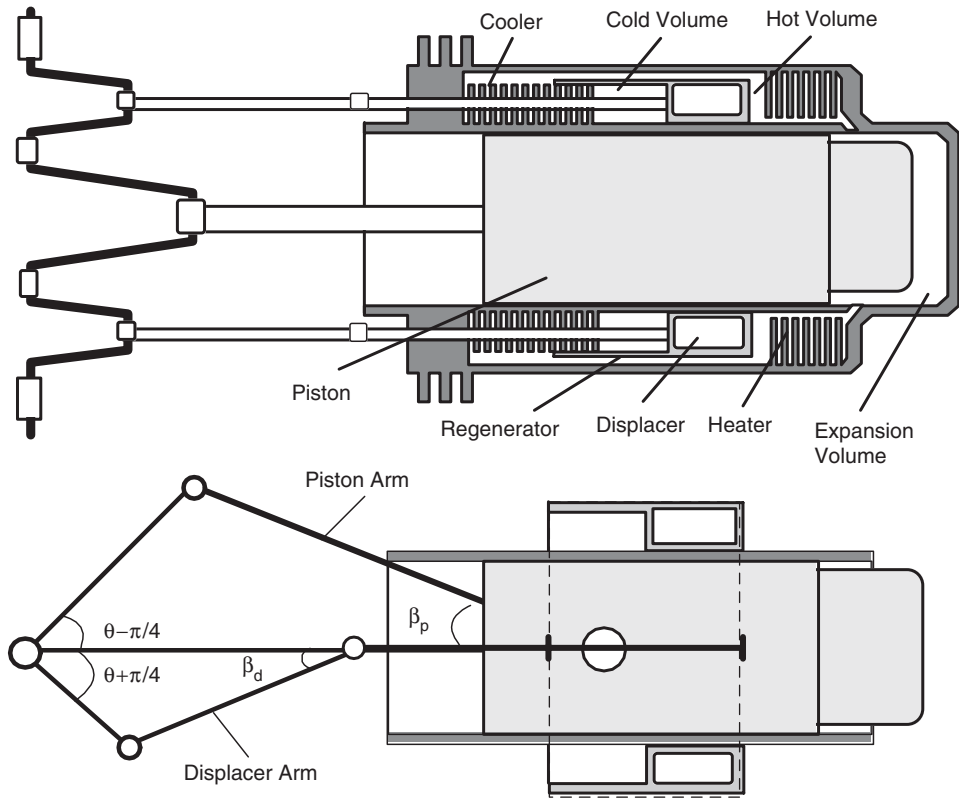


Fig. 1. Schematic illustration of the mechanical arrangement.

laminar flow. Since the heat transfer through the other surfaces has an insignificant effect on the performance of the engine, the convective heat transfer coefficient obtained for the slot surfaces is used for all the heat transfer surfaces.

The starting point of the cycle may be assumed from the positions of the displacer and power piston seen in Fig. 1, where the power piston is at 45° before its own top dead center and the displacer is at 45° after its own top dead center. At this position of the driving mechanism, the crankshaft angle θ is assumed as zero. With the first 90° rotation of the crankshaft, the power piston is confined to the top end of its own stroke, the displacer moves from up to down and the working fluid is displaced from the cold volume to the hot volume so that the working fluid undergoes a constant volume heating process while flowing through the regenerator. With the second 90° rotation of the crankshaft, while the displacer is confined to the bottom end of its own stroke, the power piston moves from up to down and an isothermal expansion process takes place. With the third 90° rotation of the crankshaft, while the power piston is confined to the bottom end of its own stroke, the displacer moves from down to up and one portion of the working fluid is displaced from the hot volume to the cold volume, as the second portion remains in the expansion cylinder, so that one portion of the working fluid experiences a constant volume cooling process and the pressure in the system is reduced. With the last 90° rotation of the

crankshaft, while the displacer is confined to the top end of its own stroke, the power piston moves from down to up and the working fluid in the expansion cylinder is pumped into the cold volume below the displacer and the cycle is completed. The last process is assumed to be a compression process at constant temperature. Thus, in the third process, not all but only one portion of the working fluid is cooled. Therefore, the cycle performed by this driving mechanism differs from the theoretical Stirling cycle.

To calculate the instantaneous values of hot volume, cold volume, expansion volume, heat transfer area in hot volume, heat transfer area in cold volume and heat transfer area in expansion cylinder, the kinematic relations,

$$s_h = (R_d + \ell_d) - R_d \cos\left(\theta + \frac{\pi}{4}\right) - \ell_d \cos \beta_d, \quad (1)$$

$$s_c = (R_d - \ell_d) + R_d \cos\left(\theta + \frac{\pi}{4}\right) + \ell_d \cos \beta_d, \quad (2)$$

$$s_e = (R_p + \ell_p) - R_p \cos\left(\theta - \frac{\pi}{4}\right) - \ell_p \cos \beta_p, \quad (3)$$

are used. In these equations, β_d and β_p are angles made by the displacer arm and the piston arm with cylinder axis, respectively, and are illustrated in Fig. 1. The relations between crankshaft angle θ and β_d , β_p are

$$\beta_d = \arcsin\left(\frac{R_d}{\ell_d} \sin\left(\theta + \frac{\pi}{4}\right)\right), \quad (4)$$

$$\beta_p = \arcsin\left(\frac{R_p}{\ell_p} \sin\left(\theta - \frac{\pi}{4}\right)\right). \quad (5)$$

The relation between crankshaft angle and time is $\theta = \omega t$. The instantaneous pressure in the system is calculated by

$$p = \frac{m_t R}{\sum_{i=1}^n V_i / T_i}. \quad (6)$$

To calculate the variation of temperature, within a nodal volume between two steps of time, the first law of thermodynamics given for the unsteady open system,

$$\Delta T_i = (h_i A_i (T_w - T)_i \Delta t - \Delta m_i C_v T + E_i - P \Delta V_i) / (m_i C_v), \quad (7)$$

is used, where E_i is the enthalpy flow in or out of the nodal volume and the approximation

$$E_i = -C_p \frac{T_i + T_{i+1}}{2} \sum_{j=i+1}^n \Delta m_j - C_p \frac{T_{i-1} + T_i}{2} \sum_{j=1}^{i-1} \Delta m_j \quad (8)$$

can be used. If $\sum_{j=i+1}^n \Delta m_j$ has a negative value, enthalpy flow occurs from the nodal volume $i+1$ to the nodal volume i ; else, the enthalpy flow occurs from the nodal volume i to the nodal volume $i+1$. Similarly, if $\sum_{j=1}^{i-1} \Delta m_j$ has a negative value, enthalpy flow occurs from the nodal volume $i-1$ to the nodal volume i ; else, enthalpy flow occurs from the nodal volume i to the nodal volume $i-1$. Instantaneous values of mass, m_j , in nodal volumes are calculated using the general state equation of perfect gases.

3. Results and discussion

Results depicted in Fig. 2 are obtained by varying the regenerator area from 0 to 0.5 m^2 while the other inputs are constant, i.e., 0.5 g air mass, 0.3 m^2 heater area, 0.15 m^2 cooler area, $250 \text{ W m}^{-2} \text{ K}^{-1}$ heat transfer coefficient, 350 K cooler temperature and 1000 K heater temperature. The volumes of the heater, regenerator and cooler are also kept constant so that the compression ratio is constant.

Theoretically, the sum of heats received and delivered by the regenerator throughout the cycle should be zero. From this case study, however, we see that the heats received from the regenerator and delivered to the regenerator are not equal to each other. Depending on the other inputs of the cycle, the difference of regenerative heat becomes sometimes positive, sometimes negative. It can also be made zero by arranging the other parameters. In practice, however, an engine may not work permanently at the same conditions. There will always be a difference between heats delivered and received. For the regenerator to be held at the same temperature distribution, its hot end should be heated, and its cold end should be cooled by neighboring elements heater and cooler. If the difference is a positive magnitude, the regenerator will receive heat from the heater. If the difference is a negative magnitude, the regenerator will give heat to the cooler.

In Fig. 2, we observe that as the regenerator area increases, the heat exchange in the heater decreases. This is partially due to the positive heat from the regenerator to the working fluid. Initially, as the regenerator area increases, the thermal efficiency decreases followed by a steady increase. The precise cause of this initial decrease is not known; however, it may be due to a numerical error. After a certain value of the regenerator area, the increase in thermal efficiency and work per cycle becomes insignificant. Therefore, any further increase in the regenerator area would not be useful, and it could increase the dead volume with adverse effects.

Results depicted in Fig. 3 are obtained by varying the working fluid mass from 0.5 to 30 g while the other inputs are constant, i.e., 0.3 m^2 regenerator area, 0.3 m^2 heater area, 0.15 m^2 cooler area, $250 \text{ W m}^{-2} \text{ K}^{-1}$ heat transfer coefficient, 350 K cooler temperature and 1000 K heater temperature.

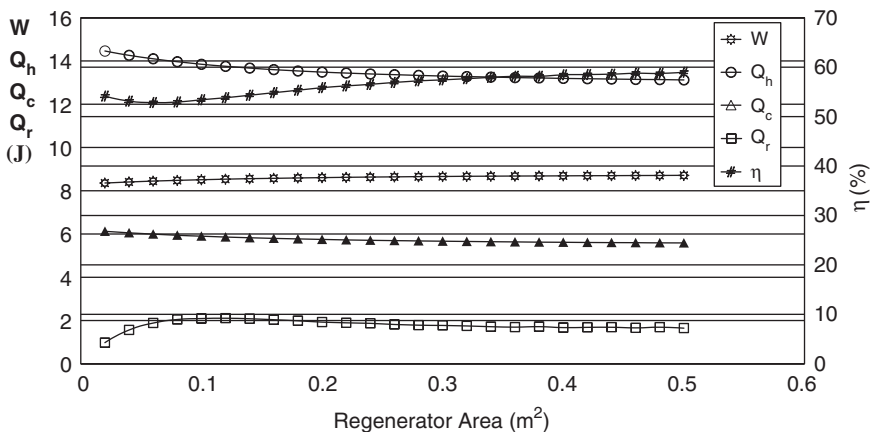


Fig. 2. Variation of work, heat from hot source, heat to cold source, regenerative heat difference and thermal efficiency with regenerator area.

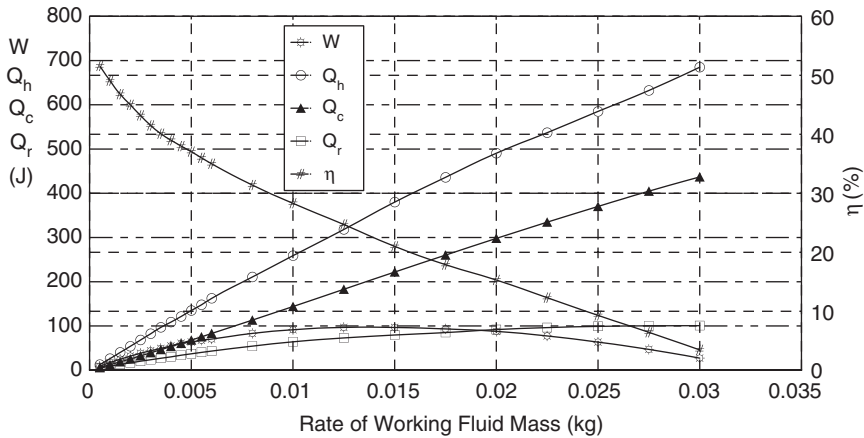
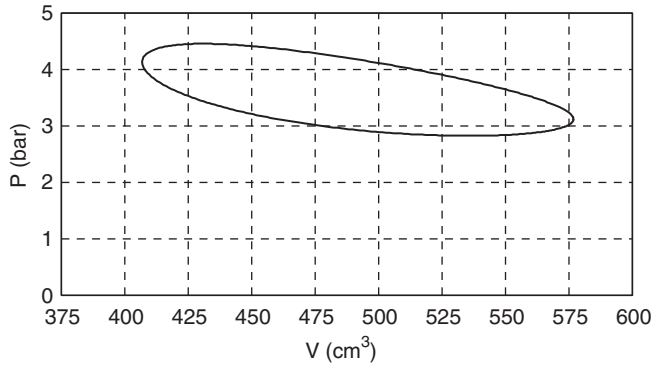
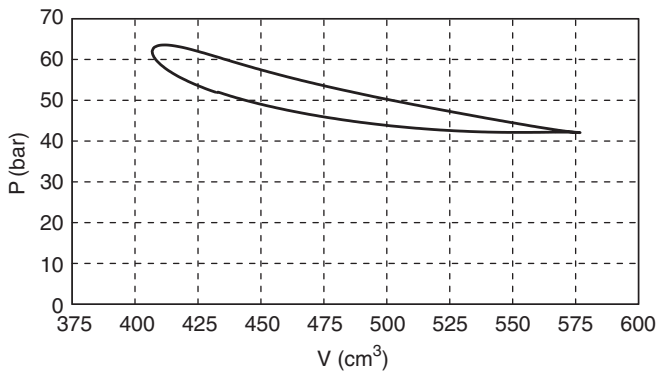


Fig. 3. Variation of work, heat from hot source, heat to cold source, regenerative heat difference and thermal efficiency with rate of working fluid mass.

As the mass of the working fluid increases, the work per cycle decelerates until the mass of the working fluid is 13 g. At 13 g working fluid mass, the maximum work per cycle obtained is 96 J. Further increase in working fluid mass decreases the work. This is due to the decrease in thermal efficiency. Compared to the mass of the working fluid, the thermal efficiency decreases steadily. At lower values of working fluid mass, the temperature of the working fluid approaches the hot source temperature after the heating process. Similarly, the working fluid temperature approaches the cold source temperature after the cooling process. As a result of this situation, the thermal efficiency approaches the Carnot efficiency. When the mass of working fluid increases, however, the fluid temperature remains lower than the hot source temperature after the heating processes. Similarly, it remains higher than the cold source temperature after the cooling processes. As a result of this situation, the thermal efficiency decreases. Under the same conditions, isothermal analysis gives about 280 W work per cycle, where the efficiency of the cycle is equal to the Carnot cycle.

In Fig. 3, we also see the variation of heat exchange in the heater, in the cooler and the regenerative heat difference. Heat exchanges in the heater and cooler increase almost linearly with the working fluid mass. The regenerative heat difference also increases, but it is limited.

In Figs. 4 and 5, p - V diagrams are shown for 1 and 13 g working fluid mass. Other inputs of Figs. 4 and 5 are the same as that of Fig. 3. At 1 g working fluid, 48% thermal efficiency and 17 J work are obtained, and at 13 g working fluid, 24% thermal efficiency and 97 J work are obtained. As the p - V diagram in Fig. 4 resembles an oval shape, the p - V diagram in Fig. 5 has a shape resembling a hydrofoil. At the beginning of the constant volume heating process, all the working fluid is in the cold volume of the displacer cylinder. During the heating process at constant volume, all the working fluid is displaced from the cold volume to the hot volume of the displacer cylinder. Therefore, all the working fluid is heated from the cold source temperature to the hot source temperature. On the other hand, at the beginning of the constant volume cooling process, some of the working fluid is in the hot volume of the displacer cylinder as the other portion is in the expansion cylinder.

Fig. 4. p - V diagram for 1 g working fluid mass.Fig. 5. p - V diagram for 13 g working fluid mass.

During the constant volume cooling process, the portion in the hot volume of the displacer cylinder is displaced from the hot volume to the cold volume of the displacer cylinder and cooled. The portion in the expansion cylinder remains in the same volume and uncooled. Therefore, in practice, constant volume heating and cooling processes are not symmetrical. This is because the driving mechanism is not able to perform the thermodynamic processes of the idealized Stirling cycle and, as a result, the p - V diagram of the practical Stirling cycle appears as in Fig. 5. Once the heat transfer surface in the cold volume of the displacer cylinder is enlarged, a partial improvement is seen. On the other hand, a larger heat transfer surface requires a larger dead volume in cold space and the performance of the engine does not improve.

In Fig. 6, the variation of gas temperature in the expansion volume, hot volume, heater and cold volume are illustrated versus the crank angle. In the same figure, the variation of expansion volume magnitude is also illustrated to decide the position of the power piston and displacer.

During the cycle, the expansion volume temperature varies between 799 and 832 K. While the crank angle varies from 0° to 45° , the power piston moves towards the top dead center of its own stroke and displaces the working fluid from the expansion cylinder to

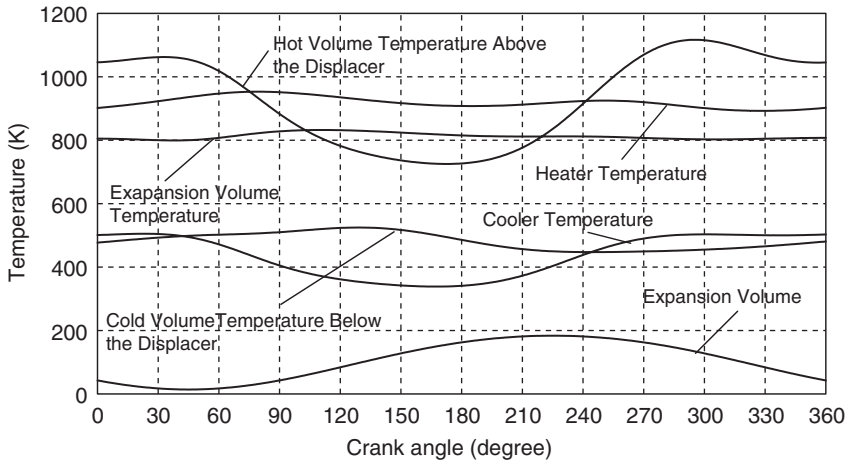


Fig. 6. Variation of work, heat from hot source, heat to cold source, regenerative heat difference and thermal efficiency with crank angle.

heater. Since the wall temperature of the expansion cylinder is higher than the temperature of the fluid flowing into the expansion cylinder, we may expect an increase in fluid temperature; however, no increase is observed. However, we observe a 4°C decrease in fluid temperature. This contradiction is caused by the outflow of enthalpy from expansion volume. In reality, the mass flowing out of the expansion volume conveys an enthalpy $\Delta m C_P T_i$. However, in this analysis, the same enthalpy is calculated as $\Delta m C_P (T_i + T_{i-1}/2)$, where $T_{i-1} > T_i$. During a mass flow into the expansion volume, we may expect an enthalpy inflow higher than the real situation. The same disadvantage of this numerical approach is valid for some of the other nodal volumes. In the case of further increasing the number of nodes, this disadvantage is avoided. The mass in the expansion volume begins to increase again after 45° crank angle, and due to the enthalpy inflow, the temperature in the expansion volume increases. The maximum temperature of expansion volume appears between 90° and 120° crank angle. The temperature shows a small decrease beyond 120° crank angle due to conversion of internal energy to work. In this configuration of the Stirling engine, during the expansion process due to enthalpy inflow to the expansion volume, the temperature remains almost constant.

During the cycle, the fluid temperature in the hot volume above the displacer varies between 1062 and 725 K. The fluid temperature in the hot volume has two maximums and one minimum. One of these maximums occurs at about 45° crank angle where the flow of working fluid is from cold volume to hot volume through the regenerator. During the same period, there is also mass flow from the expansion volume to the hot volume through the heater. Upon entering the hot volume, the fluid temperature is approximately the wall temperature and due to compression, in the hot volume, the fluid temperature exceeds the wall temperature. The second maximum occurs at about 280° crank angle where the flow of the working fluid is from the expansion volume to the cold volume through all the other volumes. When entering the hot volume, the temperature of the fluid is again approximately the wall temperature and after entering the hot volume, the fluid experiences a compression process so that its temperature exceeds the wall temperature.

The minimum occurs at about 180° crank angle where the expansion process ends. During the expansion process, the working fluid in the hot volume experiences an expansion and its temperature decreases.

The temperature of fluid in the heater varies between 952 and 892 K. The variation of fluid temperature versus the crank angle is consistent with expectations; however, processes occurring simultaneously in the heater are too complex to draw any conclusions. The temperature in the cold volume varies between 525 and 447 K. The decrease and increase in temperature occurs, respectively, in the expansion and compression period and is consistent with our expectations. The temperature in the cooler varies between 505 and 338 K. The minimum temperature appears at the end of the expansion process as 338 K. The maximum temperature is seen at the end of the flow period from the hot volume to the cold volume as 505 K, which is the result of enthalpy inflow to the cooler.

4. Conclusion

The regenerator of Stirling engines may supply some extra heat to the working fluid other than the heat received from the working fluid or vice versa. In order to balance this extra heat, the regenerator should be connected with a cooler and a heater. In nodal volumes outside the regenerator, the wall temperature and fluid temperature significantly differ from each other. If the heater is placed between the expansion volume and hot volume of the displacer cylinder, the fluid temperature in the expansion volume becomes almost constant. At low values of charging mass, higher thermal efficiency, and at high values of charging mass, lower thermal efficiency are obtained. As the working fluid mass is increased, first, the work per cycle increases, and after a certain value of increase in mass, the work per cycle tends to decrease again. As the rate of working fluid mass increases, the thermal efficiency decreases steadily starting from the Carnot efficiency.

References

- [1] Kongtragool B, Wongwises S. A review of solar-powered Stirling engines and low temperature differential Stirling engines. *Renew Sustain Energy Rev* 2003;7(2):131–54.
- [2] Dunstan D, Probert D. Raising the effectiveness of electricity generation (per unit of fossil-fuel combusted) by less conventional means. *Appl Energy* 2002;73(2):103–38.
- [3] Green MA. Recent developments in photovoltaics. *Sol Energy* 2004;76(1–3):3–8.
- [4] Bean JR, Diver RB. The CPG 5-kWe dish-Stirling development program. In: *Proceedings of the IECEC*, San Diego, Paper no 929181, 1992.
- [5] Walker G. *Stirling engines*. Oxford: Clarendon Press; 1980.
- [6] Badescu V. Optimum operation of a solar converter in combination with a Stirling or Ericsson heat engine. *Energy* 1992;17(6):601–7.
- [7] Çınar C, Karabulut H. Manufacturing and testing of a gamma type Stirling engine. *Renew Energy* 2005;30(1):57–66.
- [8] Kongtragool B, Wongwises S. Investigation on power output of the gamma-configuration low temperature differential Stirling engines. *Renew Energy* 2005;30(3):465–76.
- [9] Martini WR. *Stirling engine design manual*. US Department of Energy, DOE/NASA/3152-78/1, NASA CR-13518, 1978.
- [10] Finkelstein T. Insights into the thermodynamics of Stirling cycle machines. AIAA-94-3951-CP, 1994. p. 1829–34.
- [11] Urielli I, Bechowitz DM. *Stirling cycle machine analysis*. Bristol: Adam Hilger Ltd.; 1984.
- [12] Schimidt G. The theory of lehman's calorimetric machine. *Z Vereines Deutcher Ingenieure* 1871;15(1).
- [13] Ataer ÖE, Karabulut H. Thermodynamic analysis of the V-type Stirling cycle refrigerator. *Int J Refrig* 2005;28(2):183–9.